ALL SAINTS' COLLEGE

Ewing Avenue, Bull Creek, Western Australia

Year 12 Physics ATAR | Special Relativity | 2017

Time allowed: 50 minutes
Total marks available: 50
Show calculation answers to 3 significant figures

Student Name:______________________

Question 1 **(5 marks)**

Valentina Vladimirovna Tereshkova is a retired Russian cosmonaut, engineer, and politician. She is the first woman to have flown in space, having been selected from more than 400 applicants and five finalists to pilot Vostok 6 on 16 June 1963. She completed 48 orbits of the Earth in her three days in space.

(a) While orbiting the Earth in a spacecraft at a constant speed is Valentina in an inertial frame of reference? Explain your answer. (2)

Description	Marks
No	1
The spacecraft is not an inertial frame of reference is it is in circular motion, accelerating towards the Earth.	1
Total	2

(b) Considering only special relativistic effects, would she observe the clocks on Earth as running faster, slower or at the same speed as an identical clock on her spacecraft? Circle your answer and provide an explanation.

Slower Faster Same speed

Explanation (3)

Description	Marks
Slower	1
The Earth is moving relative to her spacecraft and she would therefore see events on Earth taking longer than the same events on her spacecraft. According to Special Relativity the time for events dilates on moving objects e.g. ticks on a clock. The observer in the spacecraft would therefore see the clocks on Earth running slow relative to the same clocks on the spacecraft.	1-2
Total	3

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Question 2 **(8 marks)**

An observer on a spaceship travels at a speed of 0.800 c relative to an observer at rest on Earth. The observer on the spaceship measures a time of 1.00 μs for a photon to travel from the back of their spaceship to the front of their spaceship.

(a) Calculate the length of the spaceship as measured by the observer on the spaceship. (2)

Description	Marks
$v = \frac{d}{t}$ $d = v \times t = 3 \times 10^8 \times 1 \times 10^{-6}$	1
$d = 300\ m$	1
Total	2

(b) Calculate the length of the spaceship as measured by an observer on Earth. (3)

Description	Marks
$L = L_0\sqrt{1 - \frac{v^2}{c^2}} = 300\sqrt{1 - \frac{0.8^2c^2}{c^2}}$ $= 300 \times \sqrt{1 - 0.8^2}$	1-2
$L = 180\ m$	1
Total	3

(c) Calculate the time that it takes the photon to travel from the back to the front of the spaceship as measured by an observer on Earth. (3)

Description	Marks
Let t = time for event Distance travelled by light as spacecraft moves forwards $ct = L + vt$ $t(c - v) = L$ $t = \frac{L}{c-v} = \frac{180}{0.2 \times 3 \times 10^8}$	1-2
$t = 3.00\ \mu s$	1
Total	3
Note time for return path of light $ct = L - vt$ $t(c + v) = L$ $t = \frac{L}{c+v} = \frac{180}{1.8 \times 3 \times 10^8} = 0.333 \times 10^{-6}$ Total time there and back agrees with time dilation for this event if $t_0 = 2.00\ \mu s$ t = 3.33 μs	

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Question 3 **(5 marks)**

Astronomers using NASA's Kepler Space Telescope discovered the first Earth-size planet orbiting a star in the "habitable zone". The habitable zone is the range of distance from a star where liquid water might pool on the surface of an orbiting planet. The discovery of Kepler-186f confirms that planets the size of Earth exist in the habitable zone of stars other than our sun.

Kepler-186f resides in the Kepler-186 system and the astronomers on Earth measured the star to be at a distance of 500 light-years from Earth. The star is in the constellation Cygnus which is also home to four companion planets, which orbit a star half the size and mass of our sun. The star is classified as an M dwarf, or red dwarf, a class of stars that makes up 70 percent of the stars in the Milky Way galaxy.

(a) Calculate the time, in the frame of reference of an observer on Earth, that it takes a spaceship to travel from Kepler-186f to Earth at 0.900 c. Note that 1 light year is the distance light travels in one year. (2)

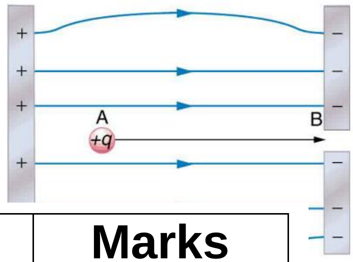
Description	Marks
$t = \frac{s}{v_{av}} = \frac{500\ ly}{0.9c}$	1
$t = 556\ years$ (1.75 x 10^{10} s)	1
Total	2

(b) Calculate the time that passes on the spaceship whilst making the journey. (3)

Description	Marks
$t_0 = t\sqrt{1-\frac{v^2}{c^2}}$ $= 556\sqrt{1-\frac{(0.9c)^2}{c^2}}$ $= 556\sqrt{1-0.9^2}$	1-2
$t_0 = 242\ years$ (7.63 x 10^9 s)	1
Total	3

Question 4 **(9 marks)**

A proton is accelerated from rest by a uniform electric field between two charged plates. The relativistic momentum of the proton after being accelerated by the charged plates is 1.03 x 10^{-18} kg m s^{-1}.



(a) Calculate the final speed of the proton. (5)

Description	Marks
$p = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}$ $\sqrt{1-\frac{v^2}{c^2}} = \frac{m_0 v}{p}$ $1-\frac{v^2}{c^2} = \frac{m_0^2 v^2}{p^2}$ $v^2\left(\frac{1}{c^2}+\frac{m_0^2}{p^2}\right) = 1$ $v = \frac{1}{\sqrt{\frac{1}{c^2}+\frac{m_0^2}{p^2}}}$ $v = \frac{1}{\sqrt{\frac{1}{(3\times10^8)^2}+\frac{(1.67\times10^{-27})^2}{(1.03\times10^{-18})^2}}}$	1-4
$v = 2.70 \times 10^8 = 0.899\,c$ (accept 0.9 c for FT)	1
Total	5

Or by insertion of data (for example)

$$1.03\times10^{-18} = \frac{1.67\times10^{-27}\times v}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{1.67\times10^{-27}}{1.03\times10^{-18}}v = 1.621\times10^{-9}\times v$$

$$1-\frac{v^2}{9\times10^{16}} = 2.6288\times10^{-18}v^2$$

$$1 = \frac{v^2}{9\times10^{16}} + 2.6288\times10^{-18}v^2 = v^2\times 1.37399\times10^{-17}$$

$$v^2 = 7.278\times10^{16} \qquad v = 2.6977888\times10^{8}\text{ m/s}$$

(b) If the proton is accelerated from rest and the final speed of the proton is 0.900 c, calculate the potential difference between the plates. Hint: You may need the equation

$$\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} = KE + m_0c^2$$

4)

Description	Marks
$KE = E - E_{rest}$ $= \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0c^2$ $= \frac{1.67\times10^{-27}\times(3.00\times10^8)^2}{\sqrt{1-\frac{(0.9c)^2}{c^2}}} - 1.67\times10^{-27}\times(3.00\times10^8)^2$	1-2
$KE = 1.95\times10^{-10}\ J$	1
$qV = KE$ $V = \frac{KE}{q} = \frac{1.95\times10^{-10}}{1.60\times10^{-19}} = 1.22\ \times\ 10^9 V = 1.22\ GV$	1
Total	4

Question 5 **(3 marks)**

A person on the website https://physics.stackexchange.com asked the following question:

"I've read in numerous places that the Large Hadron Collider is capable of accelerating protons at 0.999999991 c, which mathematically works out to being 3 metres per second slower than the speed of light. That seems so incredibly close to the speed of light, that it's hard for me to understand why we can't quite get all the way there."

Explain why the protons accelerated by the Large Hadron Collider cannot reach the speed of light.

Description	Marks
With reference to the relativistic energy equation $E = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}$ as v approaches c the denominator approaches 0. As denominator approaches zero the relativistic energy of the proton approaches infinity.	1-2
Therefore for the speed of the proton to reach c the energy added would have to be infinite which means that the proton cannot be accelerated to the speed of light. (Finite universe with finite mass and energy)	1
There are several possible explanations here, i.e. F=ma etc. Accept any reasonable explanation.	
Total	3

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Question 6 **(7 marks)**

Aneutronic fusion is any form of fusion power in which neutrons carry no more than 1% of the total released energy. Successful aneutronic fusion would greatly reduce problems associated with neutron radiation such as ionizing damage, neutron activation and requirements for biological shielding, remote handling and safety.

Some proponents see a potential for dramatic cost reductions by converting energy directly to electricity. However, the conditions required to harness aneutronic fusion are much more extreme than those required for the conventional deuterium–tritium (D-T) nuclear fuel cycle.

One example of an Aneutronic reaction is the deuterium-lithium-6 reaction

$$^{2}_{1}H + ^{6}_{3}Li \rightarrow 2^{4}_{2}He + energy$$

(a) Calculate the energy released by the above reaction. (4)

where

- mass of deuterium nucleus = 3.34358 x 10^{-27} kg,
- mass of lithium-6 = 9.98834 x 10^{-27} kg,
- mass of helium-4 = 6.64466 x 10^{-27} kg,
- mass of neutron = 1.67500 x 10^{-27} kg.

Description	Marks
$\Delta m = m_{products} - m_{reactants}$ $= (2 \times 6.64466 \times 10^{-27}) - (3.34358 \times 10^{-27} + 9.98834 \times 10^{-27})$ $= -4.26 \times 10^{-29}\ kg$	1-2
$E = \Delta mc^2$ $= -4.26 \times 10^{-29} \times (3.00 \times 10^8)^2$	1
$E = -3.834 \times 10^{-12}\ J$ Therefore energy released is $3.834 \times 10^{-12}\ J$.	1
Total	4

(b) If the energy is released as two identical photons, calculate the frequency of the photons. (3)

Description	Marks
Energy per photon $E_{ph} = E/2$ $= \frac{3.834 \times 10^{-12}}{2} = 1.917 \times 10^{-12}\ J$	1
$E_{ph} = hf$ $f = E_{ph}/h$ $= 1.917 \times 10^{-12}/6.63 \times 10^{-34}$	1
$f = 2.89 \times 10^{21}\ Hz$	1
Total	3

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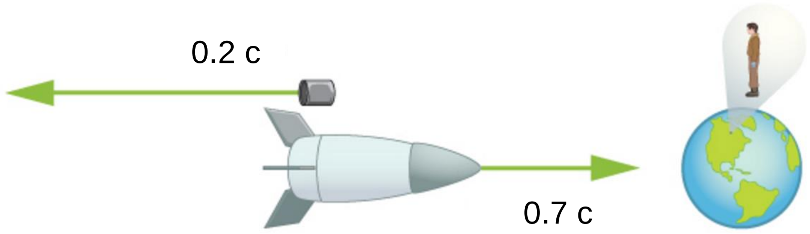
Question 7 **(7 marks)**

A spaceship travels towards an observer on Earth at a speed of 0.700 c. As the spaceship approaches Earth it ejects a weather satellite. The velocity of the satellite relative to the observer on Earth is 0.200 c away from the Earth.



(a) Calculate the velocity of the satellite relative to an observer the spaceship. (3)

Description	Marks
$u' = \frac{u-v}{1-\frac{vu}{c^2}} = \frac{-0.200c-0.700c}{1-\frac{0.700c \times -0.200c}{c^2}}$	1
$u' = \frac{-0.9.c}{1+0.14}$	1
$u' = -0.789\ c\ (2.37\ \times\ 10^8\ m\ s^{-1})$ Direction = away from ship	1
Total	3

(b) An observer on the spaceship measures the depth of the Earth's atmosphere to be 7.1 km. Calculate the depth of the atmosphere as measured by an observer on the weather (4)

Description	Marks
Reference frame spaceship (space is moving past) $L_0 = \frac{L}{\sqrt{1-\frac{v^2}{c^2}}}$ $L_0 = \frac{7.1}{\sqrt{1-0.7^2}}$ $= 9.94\ km$	1-2
Relative to the weather satellite (Earth is moving away and space is going past) $L = L_0\sqrt{1-\frac{v^2}{c^2}}$ $= 9.94\sqrt{1-0.2^2}$	1
$L =\ 9.74\ km$	1
Total	4

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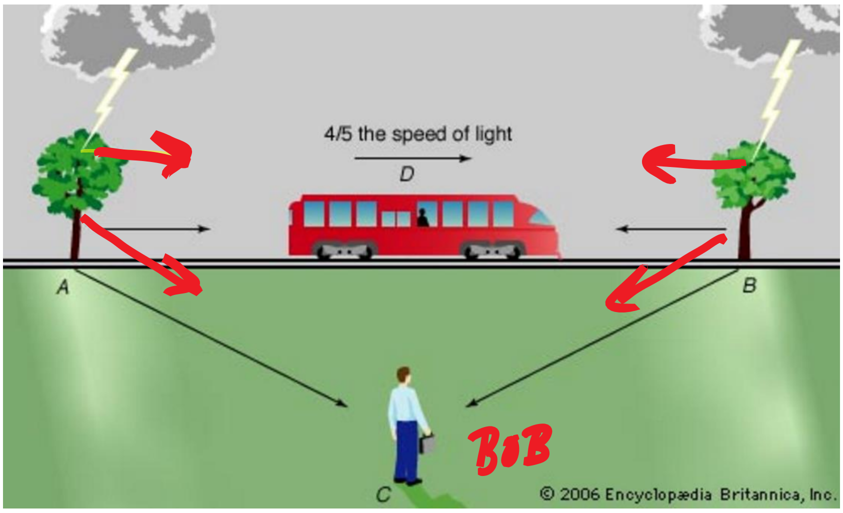
Question 8 **(6 marks)**

Alice stands in the middle of a train at Position D moving at speed 0.800 c to the right relative to Bob who is stationary on Earth and stands at Position D. When Alice is in line with Bob two bolts of lightning strike Tree A and Tree B simultaneously in Alice's frame of reference. Both Alice and Bob are midway between the trees when the lightning stuck.



(a) Does Bob see the lightning strike Tree A or Tree B first or does he see the lightning strike Tree A and B simultaneously in his frame of reference? Circle your answer and provide an explanation.

Description	Marks
Tree A	1
Alice sees both light pulses at the same time and as they travelled equal distances at equal speed the strikes were simultaneous in her frame of reference. In her frame of reference she sees Bob moving left towards the light pulse from A and away from light pulse from B. She predicts Bob will see light from tree A first (shorter distance to reach him as he moves towards it). They both agree with the sequence predictions for each other but for different reasons. (There is no spatial separation between them) In Bob's reference frame he see light from A first because it occurred first. Both light pulses travelled equal distances at the same speed in his reference frame.	1-2
Total	3

(b) Does Bob see the light from Tree A or Tree B strike Alice first or does he see the light from Tree A and B strike Alice simultaneously? Circle your answer and provide an explanation.

Description	Marks
Simultaneously	1
In Bob's frame of reference, lightning strikes Tree A first and the light from Tree A chases Alice. Light from Tree B is emitted sometime later and runs into Alice. The light from both trees hit Alice at the same time in Bob's frame of reference as light from Tree A travels for a longer time over a longer distance than light from Tree B (shorter time & distance) Also a note on spatial separation: Alice sees both lightning bolts strike at the same time as stated in the question. As she is midway along the carriage when the lightning strikes occur, light from each strike hits her at the same time. As the light hits her at the same time in the same place and there is no spatial separation between them Bob will also see the light strike Alice at the same time. (But for the reasons outlined above).	2
Total	3

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